2/3/17 "The strength to win comes from within."-Anonymous

HW: "Horizontal Stretching of functions" homework section

AIM: How do we recognize a horizontal stretch?

Warm Up:

$-2 \le y \le 8$

The function h(x) has a range (y) given by the interval [-2, 8]. The function f(x) is defined by $f(x) = \frac{1}{2}h(x) + 6$.

compression

Shift up

What is the range of f(x)?

multiply by $\frac{1}{2}$ Add 6

$-2\left(\frac{1}{2}\right) = -1 + 6 = 5$

$8\left(\frac{1}{2}\right) = 4 + 6 = 10$

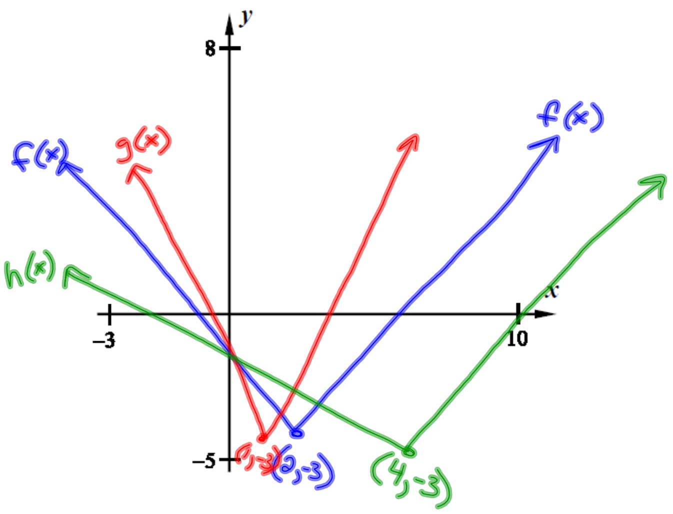
[5, 10]

|x|

***Exercise* #1:** Consider the absolute value function $f(x)=|x-2|-3$.

vertex $(2,-3)$

(a) Using your calculator, sketch a graph of f on the axes provided. Label the coordinates of its vertex point without the use of your calculator.



(b) Consider the function $g(x)=f(2x)$. Determine a formula for g and then graph it on the axes. Use your calculator to find its minimum point and label it on the graph.

$f(x)=|x-2|-3$

$g(x)=|2x-2|-3$

(c) Now consider the function $h(x)=f\left(\frac{1}{2}x\right)$. Determine a formula for h and graph it on the axes. Use your calculator to find its minimum point and label it on the graph.

$f(x)=|x-2|-3$

$h(x)=\left|\frac{1}{2}x-2\right|-3$

(d) Summarize your findings below for each function.

$f(x)$ turning point:	$f(2x)$ turning point:	$f\left(\frac{1}{2}x\right)$ turning point:
$(2,-3)$	$g(x)$ $(1,-3)$	$h(x)$ $(4,-3)$

(e) What stayed constant about the turning points? What changed and how did it change?

The y-values stayed the same.

X-value changed. We did the inverse

HORIZONTAL DILATIONS

For a real number, positive constant such that $k > 1$:

1. The function $f(kx)$ represents a horizontal compression of $f(x)$ by a factor of k (dividing by k to get x)
2. The function $f\left(\frac{1}{k}x\right)$ represents a horizontal stretch of $f(x)$ by a factor of k. (multiplying by k to get x)

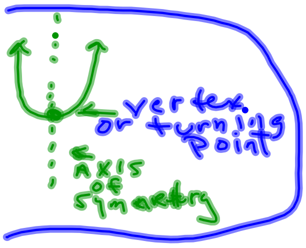
(parabola) $f(x) = ax^2 + bx + c$

***Exercise* #2:** Let's take a look at the **quadratic function** $f(x) = x^2 - 12x + 20$.

(a) Determine the coordinates of its turning point by using the equation for the axis of symmetry of $x = -\frac{b}{2a}$.

$a = 1 \quad b = -12 \quad c = 20$

$x = \frac{-(-12)}{2(1)} = \frac{12}{2} = 6$



Find y:

$f(6) = 6^2 - 12(6) + 20$

$f(6) = -16$

Turning Point: $\boxed{(6, -16)}$

(b) If g is defined by $g(x) = f(3x)$, what should be the coordinates of its turning point based on our previous work? Explain.

Compression by a factor of 3. (Dividing x-values by 3)

$6 \div 3 = 2$

$\boxed{(2, -16)}$

(c) Determine a formula for $g(x)$ and then use the turning point formula to verify your answer from part (b).

$g(x) = f(3x)$

$g(x) = (3x)^2 - 12(3x) + 20$

$g(x) = 9x^2 - 36x + 20$

$x = \frac{-b}{2a} = \frac{-(-36)}{2(9)} = \frac{36}{18} = 2$

Find y:

$g(2) = 9(2)^2 - 36(2) + 20$

$g(2) = -16$

$\boxed{(2, -16)}$

(d) Show that the y-intercept of both $f(x)$ and $g(x)$ are equal. What does this make sense from a horizontal dilation perspective?

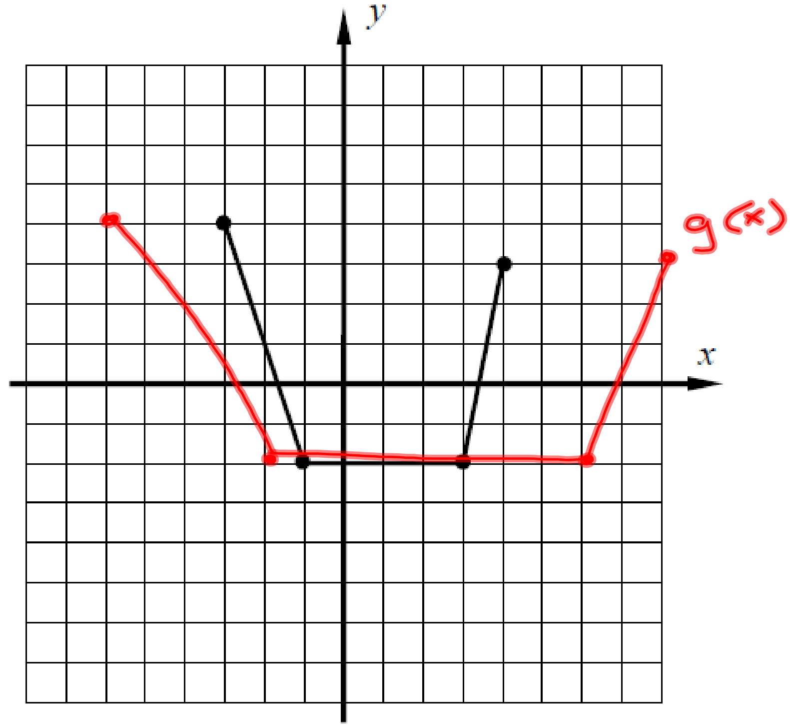
* $x = 0$ @ y-int.

$f(0) = 0^2 - 12(0) + 20 = 20$

$g(0) = 9(0)^2 - 36(0) + 20 = 20$

y-intercept is 20 for both.

***Exercise* #3:** Consider the function $f(x)$ graphed on the grid below. If $g(x)=f\left(\frac{1}{2}x\right)$ for all values of x then answer the following questions.



(a) Evaluate each of the following using the definition of g and then state the point that lies on its graph as a consequence.

$g(-6)=f\left(\frac{1}{2}(-6)\right)=f(-3)$

$g(-6)=f(-3)=\boxed{4}$

$(-6,4)$

$g(-2)=f\left(\frac{1}{2}\cdot(-2)\right)=f(-1)$

$g(-2)=f(-1)=\boxed{-2}$

$(-2,-2)$

$g(6)=f(3)=\boxed{-2}$

$(6,-2)$

$g(8)=f(4)=\boxed{3}$

$(8,3)$

(b) Graph g on the grid to the right. How would you describe its graph compared to the graph of $f(x)$?

horizontal stretch by a factor of 2.

Transformations on $f(x)$

affect y-values

$f(x) + a \longrightarrow$ up a units

$f(x) - a \longrightarrow$ down a units

$af(x) \longrightarrow$ stretch by a factor of a

$\frac{1}{a}f(x) \longrightarrow$ compress by a factor of a

$-f(x) \longrightarrow$ reflection over x-axis

affect x-values

$f(x+a) \longrightarrow$ left a units

$f(x-a) \longrightarrow$ right a units

$f(ax) \longrightarrow$ compression by factor of a

$f(\frac{1}{a}x) \longrightarrow$ stretch by a factor of a

$f(-x) \longrightarrow$ reflection over y-axis